Probability of Same Birthday

Problem: How many people do you have to have in a room before there is a 50/50 chance of two of those people having the same birthday?

You may want to try to solve this intuitively, but you will quickly run into problems. First, are there a number of people you could have that would guarantee a match? Well, as unlikely as it seems, there is still a chance that in a million people, no one was born on a particular day in the year. It looks like we will need to use some math to solve this.

What is Really Going On

There is a good chance that when we calculate the actual number, you will not believe it. This is partly because you may not be considering how many comparisons are being made. If we look at 2 people, we have 1 comparison. We check to see if these two have the same birthday and we are done. But if we add 1 more person (3 total, call them a, b, and c), suddenly we have 3 comparisons (ab, ac, bc). If we add still 1 more (called d), we now have 6 comparisons to make (ab, ac, bc, ad, bd, cd). As we add people to the room, the amount of chances of a match actually rise exponentially. There are a few ways to do this. Let’s proceed as follows:

1) What is the possibility of two people NOT having the same birthday?

$$\boldsymbol{chance\ of\ 1\ pair\ not\ matching = \frac{364}{365} = 0.99726 = 99.726\%}$$

2) Now, if we raise this to an exponential of the number of pairs of people we have, we will have the probability that there are no matches in all our people:

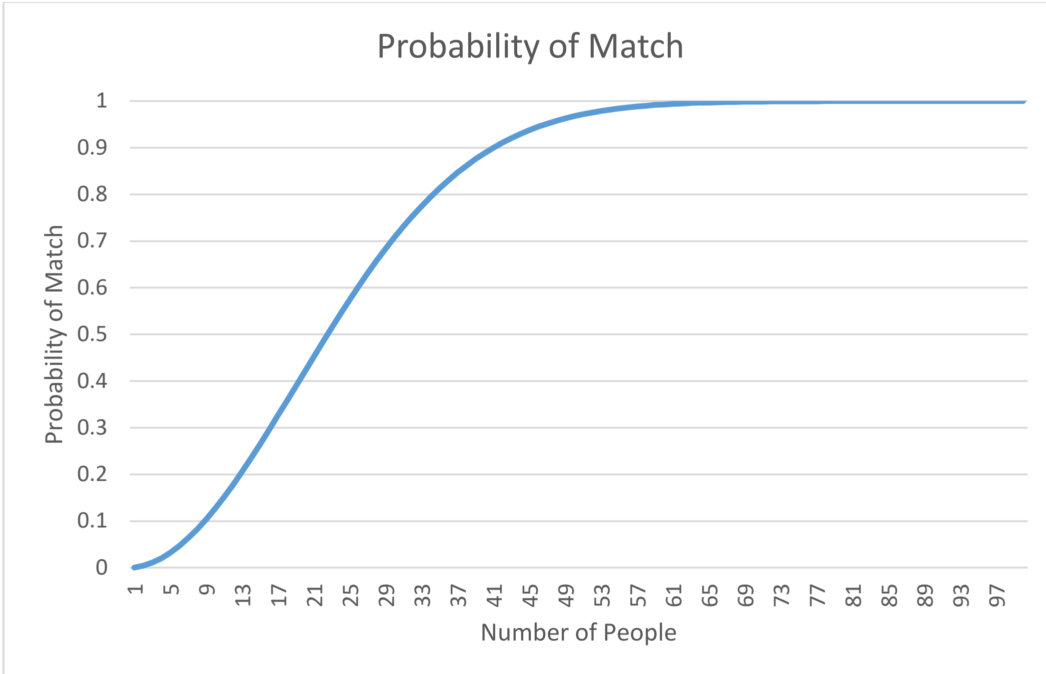
$$\boldsymbol{chance\ of\ no\ matches = 0.99726^{n(n-1)/2}}$$

where n is the number of people and n(n-1)/2 is the number of independent pairs.

3) Since we have the probability of no matches, let’s subtract it from 1 to arrive at the other possibility; that at least 1 pair matched.

$$\boldsymbol{chance\ of\ a\ match = 1 - 0.99726^{n(n-1)/2}}$$

Let’s plot this function to see where we cross the 50% line:



If we look at the 0.5 (50%) line on the y-axis, it intercepts the function at something around 23. Could this be right? We only need 23 people in a room to get a 50/50 chance that two people will have the same birthday? Yes. The math doesn't lie. The secret is in the number of pairs. For the 23 people, let's calculate the number of independent people pairs we have:

$$independant\ pairs = \frac{23(22)}{2} = \mathbf{253}$$

That means that in a room of 23 people, there are actually 253 independent checks I can do to look for the same birthday. With only 365 possibilities for birthdays and 253 independent checks, it becomes a little easier to see why we can reasonable expect a match with such a small number of people.